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Abstract. The electrostatic self-focusing mechanism for electron streams is shown to be a stronger focusing process than previously realized in that most of the potential drop that obtains between the stream axis and the surrounding tube wall occurs in a rapid potential variation in and near the stream itself. Streams are self-focusing when the particles have transverse energy spreads less than a well defined value that depends on the stream particle energy and the choice of ambient gas. The stream diameter is inversely proportional to the current for low values of stream perveance, but is independent of the current for high values of this parameter. The average random energy of the ambient electrons is high in the stream vicinity as compared with values near the tube wall.

1. Introduction

Recent experimental results on electrostatically self-focusing electron streams (Kofoid and Zieske 1965, Cabral *et al* 1969, Jancarik *et al* 1969, McCorkle and Bennett 1971) indicate that an adequate model must recognize the influence of (*a*) the transverse energy spread of the stream particles, (*b*) the production of relatively high energy ambient electrons due to stream-particle–neutral ionizing collisions, (*c*) the loss of relatively low energy ambient electrons to the tube wall, (*d*) the transmitted electron flux near the tube wall and (*e*) charge exchange collisions between the ions produced by the stream and the ambient neutrals. The description given below is that of a selfconsistent model for the stream distribution and the potential profile in and around the stream vicinity, as well as a description of sheath formation near the tube wall. Ambient charged particle distributions and the average energy of each type of charged particle are specified, the formulation being consistent with energy transport principles.

2. Definition of electrostatic self-focusing of electron streams

Electrostatic self-focusing of an electron stream results from the establishment of radial electric fields in the vicinity of the stream due to a net electrostatic charge accumulation in that region. A redistribution of the ionization produced by stream-particle–neutral collisions gives rise to this space charge. These fields, which focus the stream, simultaneously eject positive ions from the beam region.

The steady state equation for the radial disposition of stream electrons interacting electromagnetically with themselves as well as other charged particles present has been given (Bennett 1955). For radially symmetric and longitudinally uniform streams, the first moment with respect to radial velocity of the Boltzmann equation for stream electrons was shown to be

$$\frac{\partial \dot{r}_{\mathrm{b}0}}{\partial t}+\dot{r}_{\mathrm{b}0}\frac{\partial \dot{r}_{\mathrm{b}0}}{\partial r}-\frac{F_{\mathrm{r}}}{m}-\frac{2\theta_{\mathrm{b}}}{mr}+\frac{1}{rn_{\mathrm{b}}}\frac{\partial}{\partial r}\left(rn_{\mathrm{b}}\frac{2\psi_{\mathrm{b}}}{m}\right) = 0 \tag{1}$$

where n_{b} is the stream electron particle density, ψ_{b} is the mean kinetic energy of these particles due to radial motion relative to the radial velocity of mass motion, $\dot{r}_{\mathrm{b}0}$; θ_{b} is the mean kinetic energy of these particles due to tangential motion, m is the electronic mass and F_{r} is the radial component of force arising from the interaction of all particles in the stream vicinity. For electrostatically pinched streams ($v_{\mathrm{b}} \ll c$), the magnetic interaction may be neglected. The radial electrostatic force on a stream electron at position r is

$$F_{\mathrm{r}} = -\frac{e}{\epsilon_0 r}\sum_{\alpha} e_{\alpha}\int_0^r n_{\alpha}(s)s\,\mathrm{d}s$$

in mks units, where n_{α} is the particle density of type α particles, the summation extending over all types present. Utilizing this force expression in equation (1), taking the derivative with respect to the radius and multiplying by $2/r$ gives

$$\frac{1}{2}m\frac{\partial}{\partial r}\left\{r\left(\frac{\partial \dot{r}_{\mathrm{b}0}}{\partial t}+\dot{r}_{\mathrm{b}0}\frac{\partial \dot{r}_{\mathrm{b}0}}{\partial r}\right)\right\}+\frac{\chi_{\mathrm{b}}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \ln n_{\mathrm{b}}}{\partial r}\right) = -\frac{e^2}{\epsilon_0}(n_{\mathrm{i}}-n_{\mathrm{e}}-n_{\mathrm{b}}) \tag{2}$$

where n_{i} and n_{e} are the ambient ion and electron particle densities, respectively, and $\theta_{\mathrm{b}} = \psi_{\mathrm{b}} \equiv \frac{1}{2}\chi_{\mathrm{b}}$ has been taken as a constant. Only singly charged ions are considered. In the following treatment, uniform diameter streams, $\dot{r}_{\mathrm{b}0} = 0$, are studied.

In an analogous manner, the radial potential development in and around an electrostatically pinched electron stream of uniform diameter may be described by Poisson's equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) = -\frac{1}{\epsilon_0}\sum_{\alpha} e_{\alpha}n_{\alpha}. \tag{3}$$

Consistency between equations (2) and (3) requires that

$$n_{\mathrm{b}} = n_{\mathrm{b}0}\exp\left(\frac{e\phi}{\chi_{\mathrm{b}}}\right) \tag{4}$$

thus specifying the steady state stream particle distribution.

For purposes of estimation, parameters in the following range are considered: beam currents from the order of hundreds of microamperes to tens of milliamperes, beam voltages from approximately 100 V to several kilovolts, ambient gas pressures from approximately 10^{-5} Torr to 10^{-2} Torr, beam radius of several millimetres, tube radius of several centimetres, beam length of approximately a metre, typical ambient gas—mercury vapour.

3. Charge densities produced in the electrostatic pinch

Positive ions, having received negligible energy during the ionization process, emerge radially from the stream region under the influence of the electric field and experience insignificant collisional interaction in their passage to the tube wall, except for possible resonance charge exchange collisions with parent gas particles (Kovar 1964, McConnell and Moiseiwitsch 1969, McCorkle and Bennett 1971). Charge exchange is of importance for the higher pressure values considered here for beam operation in mercury vapour.

Solution of the Boltzmann equation for the ion particle density in and around the stream under these conditions gives (see appendix 1)

$$n_i(r) = \frac{1}{r}\int_0^r \mathrm{d}s(\nu_i n_b(s)s + \psi(s)) \exp\{n_n B(r|s)\}\left(\frac{2e}{M}(\phi(s)-\phi(r))\right)^{-1/2} \tag{5}$$

where

$$\psi(r) = n_n \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \dot{r}\sigma_{ex}(\dot{r}^2) f_i(r, \dot{r}, \dot{\theta}, \dot{z}) r \,\mathrm{d}\dot{r}\,\mathrm{d}\dot{\theta}\,\mathrm{d}\dot{z}$$

and

$$B(r|s) = \int_r^s \sigma_{ex}\left(\frac{2e}{M}(\phi(s)-\phi(x))\right) \mathrm{d}x$$

in which ν_i is the ionization frequency (ionization being attributed to stream-electron–neutral collisions exclusively), n_n is the neutral particle density each atom of which presents a charge exchange cross section of $\sigma_{ex}(\dot{r}^2)$ dependent upon the ion energy attributable to radial motion, M is the ion mass, and $f_i(r, \dot{r}, \dot{\theta}, \dot{z})$ is the ion distribution function. In the limit of negligible charge exchange, $\psi(s) \to 0$ and $B(r|s) \to 0$, the ion particle density is

$$n_i(r) = \frac{1}{r}\int_0^r \mathrm{d}s\; \nu_i n_b(s)s\left(\frac{2e}{M}(\phi(s)-\phi(r))\right)^{-1/2} \tag{6}$$

a result readily obtainable from the continuity equation.

In contrast with the ions, ambient electrons emerge from the ionization process with a considerable spread of energies. Of interest is the average energy E_0 acquired by the ejected electron during the collision. In the classical approximation (Vriens 1966), this value is

$$E_0 = \frac{\Sigma_k n_k (E_1 + E_{2k} + U_{ik})^{-1}\{\ln(E_1/U_{ik}) - 2E_{2k}/3E_1 - 1 + U_{ik}/E_1 + 2E_{2k}U_{ik}/3E_1^2\}}{\Sigma_k n_k (E_1 + E_{2k} + U_{ik})^{-1}(1/U_{ik} - 1/E_1 + 2E_{2k}/3U_{ik}^2 - 2E_{2k}/3E_1^2)} \tag{7}$$

where the incident electron has energy E_1, the interacting atomic electrons in state k number n_k per atom and have a kinetic energy E_{2k} and an ionization energy U_{ik}. (For stream energies of hundreds of electron volts in mercury vapour, only the two 6s and ten 5d atomic electrons are likely to interact collisionally.)

From a consideration of collision time scales and ambient electron lifetimes, the outward flux of electrons in the region surrounding the stream may be described as a drift motion, occurring at a characteristic speed v_e, superimposed on a random motion, the energy for which is written as $\frac{3}{2}\chi_e(r)$, and where the particle density is taken as

$$n_e(r) = n_{e0} \exp\left(\frac{e\phi(r)}{\chi_e(r)}\right). \tag{8}$$

The truncation of this distribution that occurs near the tube wall and which may extend some distance into the ambient medium is considered below in a discussion of wall–sheath formation. The random energy term may be written as

$$\chi_e(r) \simeq \tfrac{2}{5}(E_0 + e\phi(r)) \tag{9}$$

from a consideration of particle conservation and energy transport (see Appendix 2).

4. Steady state solutions and stationary conditions

A majority of the ions undergoes free flight motion over most of the pressure range considered here. Steady state solutions are obtained in this work for cases in which the ion source term may be written as in equation (6). For uniform diameter streams, the stream development is specified by

$$\frac{\chi_b}{r}\frac{d}{dr}\left(r\frac{d\ln n_b}{dr}\right) = -\frac{e^2}{\epsilon_0}\left\{\frac{v_i}{r}\int_0^r \frac{n_b(s)s\,ds}{\{2e(\phi(s)-\phi(r))/M\}^{1/2}} - n_{e0}\exp\left(\frac{5e\phi}{2E_0+2e\phi}\right) - n_b\right\}. \tag{10}$$

The beam model, $n_b = n_{b0}(1+r^2/\sigma^2)^{-2}$, where the nominal stream radius is given as

$$\sigma = \left(\frac{m}{M}\frac{2\chi_b}{eV_b}\right)^{1/2}\frac{K(\alpha)}{P_i p}\left(1+\frac{8\pi\epsilon_0\chi_b}{N_b(\infty)e^2}\right) \tag{11}$$

for which

$$K(\alpha) = \frac{\exp\{-5\alpha\ln 2/(1-2\alpha\ln 2)\}-\frac{1}{4}}{\exp\{-5\alpha\ln 2/(1-2\alpha\ln 2)\}-e^{-1}D(1)}$$

where

$$\alpha \equiv \chi_b/E_0 \qquad \text{and} \qquad D(1) = \int_0^1 e^{t^2}\,dt$$

is the Dawson function evaluated at unity argument, is found to be a consistent solution of equation (10) to within the accuracy of the source term specifications (the agreement between the left and right hand sides of this equation being less than 5% throughout the beam region, $r \leqslant \sigma$). Solutions were investigated for $0 \leqslant \alpha \leqslant 0{\cdot}14$, the range of this parameter being determined from the variation of K with α (see figure 1). All finite values of stream radii are spanned for $0 \leqslant \alpha \leqslant 0{\cdot}143\,298\,1865$ since $\sigma \propto K(\alpha)$. It is

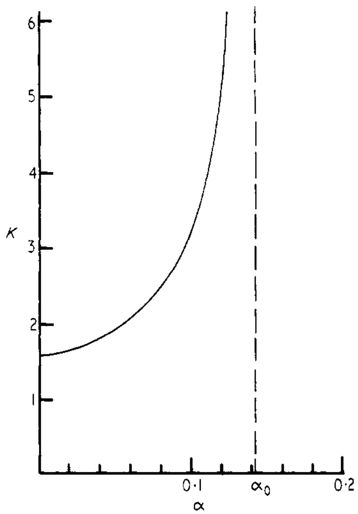

Figure 1. Representation of $K(\alpha)$. The asymptote shown is at $\alpha_0 = 0{\cdot}143\,298\,1865$.

apparent that this finite range of α values corresponds to stream angular divergences at crossover being less than some maximum value.

Outside the stream, $\Sigma_\alpha e_\alpha n_\alpha = -(\epsilon_0 \chi_b/e)\nabla^2 \ln n_b$ is of $O(\sigma^4/r^4)$ and consequently approaches zero at radial positions of several beam radii. The quasineutral medium in this region may be described by the plasma approximation, $\Sigma_\alpha e_\alpha n_\alpha \simeq 0$. Employing normalized variables; $\rho \equiv r/\sigma, \eta \equiv -e\phi/2\chi_b$, the solutions inside and outside the stream are given by

$$\eta = \ln(1+\rho^2)$$

$$\rho \simeq \frac{K}{K-1}\exp\left(-\eta + \frac{5\alpha\eta}{1-2\alpha\eta}\right) D(\sqrt{\eta}) \qquad (12)$$

respectively, $D(\sqrt{\eta})$ being the Dawson function. These solutions yield a one parameter set of curves since $K = K(\alpha)$. Figure 2 shows typical profiles obtained from these equations. Figure 3 presents a particular experimental case.

For relatively low perveance streams, $P < \frac{4}{3}10^4(\chi_b/eV_b)$, the stream radius is proportional to $\chi_b^{3/2}$ and I_b^{-1}; whereas for high perveance streams, $P > \frac{4}{3}10^{-4}(\chi_b/eV_b)$, $\sigma \propto \chi_b^{1/2}$ and is independent of I_b. For all cases, $\sigma \propto (m/M)^{1/2}, P_i^{-1}, p^{-1}$. Transverse spreading of the stream due to particle scattering has been neglected, an approximation good for all particle densities of interest. The ratio of the on-axis Debye length of the ambient electrons to the stream radius is

$$\Lambda_{d0} \equiv \frac{\lambda_{d0}}{\sigma} = 0{\cdot}223\{(K-1)(\alpha + 7500 I_b/V_0 V_b^{1/2})\}^{-1/2} \qquad (13)$$

for $V_0 \equiv E_0/e$. Typically, $\alpha \sim 0{\cdot}08$, $V_b^{1/2} \sim 20$ to $30\ \mathrm{V}^{1/2}$, $V_0 \sim 10$ to 20 V, $I_b \sim 1$ mA, and consequently $\sigma \sim \lambda_{d0}$.

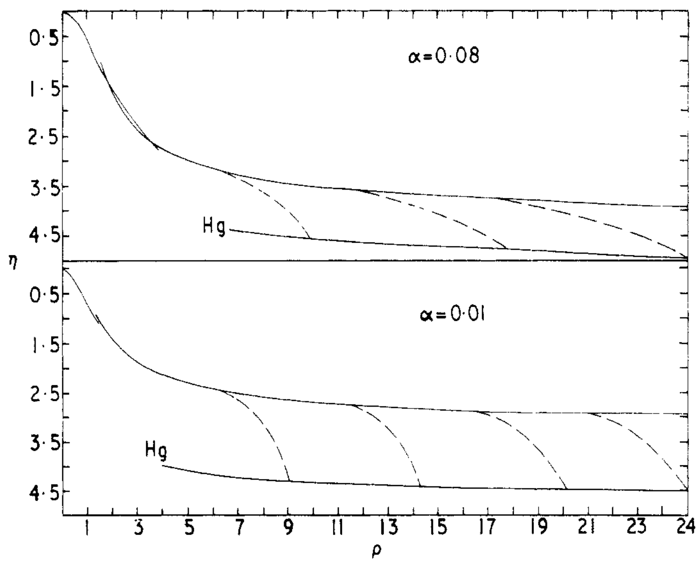

Figure 2. Representative potential profiles in and around the stream from the set of equations (12). The parameter for these curves is $\alpha = \chi_b/E_0$. Tube wall conditions (curves labelled Hg) and approximate sheath developments, as obtained from equations (17) and (18), are also shown for the case of mercury vapour as the ambient gas.

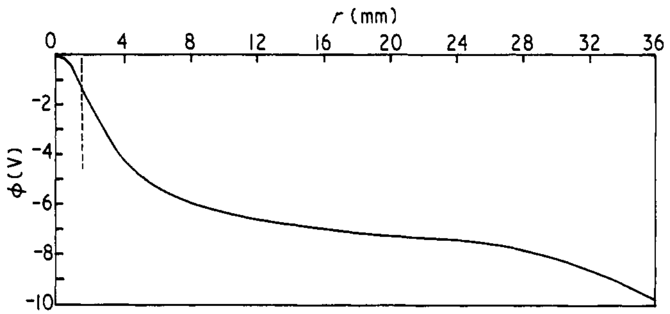

Figure 3. Potential profile to be expected for the case of a 1·6 mA, 150 V electron stream passing through mercury vapour at 5 °C, assuming $\chi_b \sim 1$ eV. The tube radius is taken to be 3·6 cm. The beam radius is indicated at about 1·5 mm, for which $\phi = -1{\cdot}4$ V. Over half (~ 5 V) of the total potential drop occurs within a distance of 5 mm of the stream axis.

5. Wall–sheath formation

At the lower pressure values considered here and at low beam currents, electrons sufficiently energetic and properly directed may escape to the tube wall from deep within the active volume and thus truncation of the electron velocity distribution tends to occur over much of the tube volume. Since the effective mechanism of energy exchange in these cases is through the coulombic selfinteraction of the electrons, the high velocity tail of the distribution is populated by a diffusion in velocity space (Parker and Tidman 1958) which results in a current to the wall that may be estimated using a Fokker–Planck model (McCorkle and Bennett 1971). In these cases, the separation of the medium surrounding the beam into a plasma and a wall–sheath is not valid as is evidenced by the fact that the Debye length near the wall may be shown to be of the order of the tube's radial dimension.

At higher pressures and (or) beam currents, sheath models (Andrews and Varey 1970) may be employed, the wall flux being nearly that of random flux. In this work it was shown that an adequate treatment of sheath formation must treat transmitted flux as well as reflected flux. Monotonic potential profiles are found to exist for all values of sheath voltage drop. The 'sheath criterion' depends upon the value of the sheath potential drop. Resulting sheath thicknesses are of the order of several Debye lengths (evaluated at the sheath edge).

The energy balance equation for electrons is evaluated at the wall as

$$\tfrac{1}{2}mv_e^2+\tfrac{5}{2}\chi_{ew} = e\phi_w+E_0-2\chi_b\{(\eta_w+1)\,e^{-\eta_w}-1\} \tag{14}$$

the last term on the right hand side being evaluated for the beam model discussed above. In general, the energy deposited at the wall by the average incident electron is $\tfrac{1}{2}mv_e^2+\tfrac{3}{2}\chi_{ew}$. The energy deposited at the wall by a freeflight ion (or by the ion plus neutrals that have received ion energy through charge exchange encounters) is $\chi_b\{\sqrt{\eta_w}\,e^{\eta_w}/D(\sqrt{\eta_w})-1\}$. Neglecting inelastic collision losses for cases of interest here, the energy boundary condition at the wall becomes

$$\frac{1}{2}mv_e^2+\frac{3}{2}\chi_{ew}+\chi_b\left(\frac{\sqrt{\eta_w}\,e^{\eta_w}}{D(\sqrt{\eta_w})}-1\right) = E_0 \tag{15}$$

since each ion–electron pair produced by beam–neutral collisions is given an initial kinetic energy of E_0. The demand that (15) be consistent with (14) gives

$$\chi_{\mathrm{ew}} = e\phi_{\mathrm{w}} - \chi_{\mathrm{b}}\left\{2(\eta_{\mathrm{w}}+1)\,\mathrm{e}^{-\eta_{\mathrm{w}}} - \frac{\sqrt{\eta_{\mathrm{w}}}\,\mathrm{e}^{\eta_{\mathrm{w}}}}{D(\sqrt{\eta_{\mathrm{w}}})} - 1\right\} \simeq \chi_{\mathrm{b}} \tag{16}$$

for the value of the electron temperature at the tube wall and

$$P \equiv \frac{R}{\sigma} \simeq \frac{K(m/M)^{1/2}\exp(2\eta_{\mathrm{w}})}{2(K-1)(1/2\alpha - \eta_{\mathrm{w}} - \frac{1}{4})^{1/2}} \tag{17}$$

as the wall condition relating the normalized wall potential η_{w} to the normalized wall position P. The normalized Debye length at the sheath edge is

$$\Lambda_{\mathrm{ds}} \equiv \frac{\lambda_{\mathrm{ds}}}{\sigma} = \frac{\exp(\eta_{\mathrm{s}})}{2}\{2(K-1)(1+7500I_{\mathrm{b}}/\chi_{\mathrm{b}}V_{\mathrm{b}}^{1/2})\}^{-1/2} \tag{18}$$

where η_{s} is the normalized potential at the sheath edge (obtainable from equation (12)). Wall potential values are shown in figure 2 for the case of mercury and reasonable sheath developments are indicated (for which Λ_{ds} was approximated by $\mathrm{e}^{\eta_{\mathrm{s}}}/4(K-1)^{1/2}$.

6. Discussion

Although self-focusing in electron streams occurs primarily through magnetic interaction at high values of stream current and voltage, for relatively low values of these parameters the predominant focusing mechanism is electrostatic. The general formulation of the electrostatic pinch given here (equations (2), (3), (5), the ambient electron model defined by equation (8) and the general energy balance requirements given in Appendix 2) provides a description of these streams over most anticipated ranges of parameters. In particular, at high gas pressures, the influence of inelastic collision processes must be recognized in the energy balance equation for the ambient electrons. Additionally, as charge exchange becomes important, the ion-density model given by equation (5) should be employed along with an appropriate cross section model (McConnell and Moiseiwitsch 1969) for this process. For streams that are somewhat removed from the stationary conditions considered here, the stream evolution (ie beam 'bouncing', phase mixing, radial loss rates, etc) is governed by equation (1). It is conjectured that heating of the plasma by the beam–plasma mechanism may be describable with the present general model by increasing the parameter E_0 as necessary for cases in which the interactions are reasonably well localized in the beam vicinity. The establishment of widespread noise and oscillations throughout the surrounding plasma would obviously invalidate the above description. In such cases, the electron distribution may be reasonably well characterized by a Boltzmann factor with a uniform temperature, and the effects of energy loss by radiation and of ionization by ambient electrons would need to be considered.

Focusing mechanisms not considered in this work are the effects of velocity modulation and the presence of the two-stream instability (Zinchenko and Zhigailo 1965, Krasovitskii 1969), as well as possible effects due to ionization oscillations (Roth 1967, 1969, Pekarek 1968, Nedospasov 1968). However, it is thought that the model presented is definitive of steady state conditions occurring in relatively quiet systems and thus provides a proper model about which the influence of perturbations may be examined.

Acknowledgments

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Appendix 1

The steady state Boltzmann equation for the ion distribution function in the case of cylindrical symmetry and longitudinal uniformity is

$$\left(\frac{F_r}{M}+r\dot{\theta}^2\right)\frac{\partial f_i}{\partial \dot{r}}-\frac{2\dot{r}\dot{\theta}}{r}\frac{\partial f_i}{\partial \dot{\theta}}+\dot{r}\frac{\partial f_i}{\partial r}=\left(\frac{\partial f_i}{\partial t}\right)_{\text{coll}}$$

when only radial forces are present. The ion particle density is given by

$$n_i(r)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_i(r,\dot{r},\dot{\theta},\dot{z})r\,\mathrm{d}\dot{r}\,\mathrm{d}\dot{\theta}\,\mathrm{d}\dot{z}.$$

The collision term is due to (*a*) the production of very nearly zero kinetic energy ions by stream electron–neutral collisions

$$\nu_i n_b(r)\delta(\dot{r})\delta(r\dot{\theta})\delta(\dot{z})$$

in which the functions indicated are Dirac delta functions, (*b*) the loss of energetic ions by resonance charge exchange with parent neutrals

$$-n_n\sigma_{ex}(\dot{r}^2)\dot{r}f_i(r,\dot{r},\dot{\theta},\dot{z})$$

and (*c*) gain of zero kinetic energy ions by this same charge exchange process at the rate

$$\psi(r)\delta(\dot{r})\delta(r\dot{\theta})\delta(\dot{z})$$

in which

$$\psi(r)=n_n\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\dot{r}\sigma_{ex}(\dot{r}^2)f_i(r,\dot{r},\dot{\theta},\dot{z})r\,\mathrm{d}\dot{r}\,\mathrm{d}\dot{\theta}\,\mathrm{d}\dot{z}.$$

Since all positive ion sources produce ions at rest and the only force present is the radial electrostatic force attributable to a variation in the potential $\phi(r)$, Boltzmann's equation becomes

$$\dot{r}\frac{\partial f_i}{\partial r}-\frac{e}{M}\frac{\mathrm{d}\phi}{\mathrm{d}r}\frac{\partial f_i}{\partial \dot{r}}=(\nu_i n_b(r)+\psi(r))\delta(\dot{r})\delta(r\dot{\theta})\delta(\dot{z})-n_n\dot{r}\sigma_{ex}(\dot{r}^2)f_i$$

the solution of which may be written as

$$f_i(r,\dot{r},\dot{\theta},\dot{z})=\frac{2\delta(r\dot{\theta})\delta(\dot{z})}{r}\int_0^r \mathrm{d}s(\nu_i n_b(s)s+\psi(s))\exp(n_n B(r|s))$$
$$\times\delta\left(\dot{r}^2-\frac{2e}{M}(\phi(s)-\phi(r))\right).$$

The ion density, as specified above, becomes

$$n_i(r) = \frac{1}{r}\int_0^r ds(v_i n_b(s)s + \psi(s))\exp(n_n B(r|s))\{2e(\phi(s)-\phi(r))/M\}^{-1/2}.$$

Appendix 2

The radial motion of ambient electrons is subject to the continuity requirement $2\pi r n_e v_e = v_i N_b(r)$ where $N_b(r)$ is the number of stream particles per unit axial distance contained within a radial distance r

$$N_b(r) = 2\pi \int_0^r s n_b(s)\, ds.$$

Energy conservation may be written as (Rose and Clark 1961)

$$\frac{1}{r}\frac{d}{dr}\left\{\frac{m}{2} r n_e \left(v_e^3 + \frac{5}{m} v_e \chi_e\right)\right\} = j_{er}E_r + \int d\boldsymbol{v}\frac{m}{2}v^2\left(\frac{\partial f_e}{\partial t}\right)_{\text{coll}}$$

Collisional energy exchange for these electrons is through energy production at the rate $v_i n_b(r) E_0$ by stream–neutral ionizing collisions, elastic and inelastic losses to neutrals (as well as possible superelastic electron–neutral events for the high pressure ranges in mercury vapour), bremsstrahlung losses, and coulombic exchanges with the ions. Except for energy production by the ionizing collisions, all collisional energy exchange terms are dropped, the estimated error in doing so being perhaps as large as 10–20% at the higher pressure values of interest (the estimate being based on data pertaining to mercury vapour). Under these restrictions, the energy balance equation becomes

$$\frac{m}{2}v_e^2 + \frac{5}{2}\chi_e = e\phi + E_0 - \frac{e}{N_b(\infty)}\int_0^r \phi(s) N_b'(s)\, ds$$

with the help of the continuity equation. The energy due to drift motion, $\frac{1}{2}mv_e^2$, may be shown to be negligibly small when compared with the energy term due to random motion, $\frac{5}{2}\chi_e$. Likewise, the last term on the right hand side of the energy balance equation is at most approximately 10% of the value of the other terms on this side of the equation. These small terms are considered to be offsetting in the equation (except for the region of wall–sheath formation) and consequently

$$\chi_e(r) \simeq \tfrac{2}{5}(E_0 + e\phi(r)).$$

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